Malaysian Journal of Mathematical Sciences 4(2): 159-169 (2010)

Flow and Heat Transfer over a Stretching Cylinder with Prescribed Surface Heat Flux

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ABSTRACT

The steady laminar flow caused by a stretching cylinder immersed in an incompressible viscous fluid with prescribed surface heat flux is investigated. The governing partial differential boundary layer equations in cylindrical form are first transformed into ordinary differential equations before being solved numerically by a finite-difference method. The problem under consideration reduces to the flat plate case when the curvature parameter is absent, and thus the results obtained can be compared with that case which is available in the literature as well as the exact solution, and found to be in good agreement. The solutions for the heat transfer characteristics are evaluated numerically for different parameters, such as the curvature parameter γ, and the Prandtl number Pr . It is observed that the surface shear stress and the heat transfer rate at the surface increase as the curvature parameter increases.

Keywords: Boundary layer, stretching cylinder, heat transfer, fluid mechanics

INTRODUCTION

The study of boundary layer flow and heat transfer resulting from stretching surface is important in manufacturing processes. Examples are numerous and they include the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films, and polymer extrusion. The quality of the final product depends on the rate of heat transfer at the stretching surface. Since the pioneering study by Crane, (1970), who presented an exact analytical solution for the steady two-dimensional flow due to a stretching surface in a quiescent fluid, many authors have considered various aspects of this problem and obtained similarity solutions. A similarity solution is one in which the number of independent variables is reduced by at least one, usually by a coordinate transformation. Despite the growth of the boundary layers with distance from the leading edge, the velocity and temperature profiles remain geometrically similar. The boundary layer flow due to a stretching vertical surface in a quiescent viscous and incompressible fluid when the buoyancy forces are taken into consideration have been considered in the papers by Chen, (1998, 2000), Ali, (2004) and Ishak *et al.*, (2007a,b, 2008a). Problems on a stretched sheet or a cylinder with prescribed surface heat flux have been introduced in many other studies (Elbashbeshy, (1998), Ishak *et al.*, (2008b), Ishak, (2009) and Bachok and Ishak, (2009a)).

The present study considers the flow and heat transfer along a stretching cylinder with prescribed surface heat flux. The technological applications include hot rolling, wire drawing and fiber production. The surface heating condition is different from those considered by Ishak and Nazar, (2009), who considered the case of a prescribed surface temperature. Analytical solutions to the wall temperature are derived in terms of the Kummer's functions (Abramowitz and Stegun, (1965)). The present study may be regarded as the extension of the papers by Elbashbeshy, (1998) and Liu, (2005), from a stretching sheet to a stretching cylinder. Thus, the results obtained can be compared with those of Elbashbeshy, (1998) and Liu, (2005), when the curvature parameter is neglected.

PROBLEM FORMULATION

Consider a steady laminar flow caused by a stretching cylinder with radius a placed in an incompressible viscous fluid of constant temperature T_∞. It is assumed that the cylinder is stretched in the axial direction with velocity $U_w(x)$ and the surface of the cylinder is subjected to a variable heat flux $q_w(x)$. Under these assumptions, the governing equations are

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right), \tag{3}$$

where x and r are coordinates measured along the surface of the cylinder and in the radial direction, respectively, with u and v being the corresponding velocity components. Further, T is the temperature in the boundary layer, ν is the kinematic viscosity coefficient and α is the thermal diffusivity. The boundary conditions are

$$\begin{aligned} &u = U_w, \quad v = 0, \quad k\frac{\partial T}{\partial y} = -q_w(x) \quad \text{at} \quad r = a, \\ &u \to 0, \quad T \to T_\infty \quad \text{as} \quad r \to \infty. \end{aligned} \tag{4}$$

We assume that $U_w(x)$ and $q_w(x)$ are of the form

$$U_w(x) = c_1\left(\frac{x}{\ell}\right), \quad q_w(x) = c_2\left(\frac{x}{\ell}\right), \tag{5}$$

where c_1 and c_2 are constants, and ℓ is a reference length scale. We look for similarity solutions of equations (1)-(3), subject to the boundary conditions (4), by writing

$$\begin{aligned} &\eta = \frac{r^2 - a^2}{2a}\left(\frac{U_w}{\nu x}\right)^{1/2}, \quad \psi = (\nu x U_w)^{1/2}\, a\, f(\eta), \\ &T = T_\infty + \frac{q_w}{k}\left(\frac{\nu x}{U_w}\right)^{1/2} \theta(\eta), \end{aligned} \tag{6}$$

where η is the similarity variable, ψ is the stream function defined as $u = r^{-1}\partial\psi/\partial r$ and $v = -r^{-1}\partial\psi/\partial x$, which identically satisfies equation (1), and k is the thermal conductivity. By defining η in this form, the boundary conditions at r = a reduce to the boundary conditions at $\eta = 0$, which is more convenient for numerical computations. From transformation (6), we obtain

$$u = U_w f'(\eta) \quad \text{and} \quad v = -\frac{a}{r}\left(\frac{\nu c_1}{\ell}\right)^{1/2} f(\eta), \tag{7}$$

where primes denote differentiation with respect to η.

Substituting (6) into equations (2) and (3), we obtain the following nonlinear ordinary differential equations:

$$(1+2\gamma\eta) f''' + 2\gamma f'' + f f'' - f'^2 = 0, \tag{8}$$

$$(1+2\gamma\eta)\theta'' + 2\gamma\theta' + Pr(f \theta' - f'\theta) = 0, \tag{9}$$

subject to the boundary conditions (4) which become

$$\begin{aligned} &f(0)=0, \quad f'(0)=1, \quad \theta'(0)=-1, \\ &f'(\eta)\to 0, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty, \end{aligned} \tag{10}$$

where γ is the curvature parameter and Pr is the Prandtl number defined respectively as

$$\gamma = \left(\frac{v\ell}{c_1 a^2}\right)^{1/2}, \qquad Pr = \frac{v}{\alpha}. \tag{11}$$

The main physical quantities of interest are the values of $f''(0)$, being a measure of the skin friction, and the non-dimensional surface temperature $\theta(0)$. Our main aim is to find how the values of $f''(0)$ and $\theta(0)$ vary in terms of the parameters γ and *Pr*.

We note that when $\gamma = 0$ (flat plate), Equation (8) reduces to those considered by Crane, (1970) where the closed-form solution is given by

$$f(\eta) = 1 - e^{-\eta}. \tag{12}$$

Further, for this case ($\gamma = 0$), the solution of Equation (9) subject to the associated boundary conditions (10) is given by

$$\theta(\eta) = \frac{1}{\Pr} e^{-\Pr\eta} \frac{M(\Pr-1, \Pr+1, -\Pr e^{-\eta})}{M(\Pr-1, \Pr, -\Pr)}, \tag{13}$$

where $M(a,b,z)$ denotes the confluent hypergeometric function (Abramowitz and Stegun, (1965)).

The wall temperature is given by

$$\theta(0)=\frac{1}{\Pr}\frac{M(\Pr-1,\Pr+1,-\Pr)}{M(\Pr-1,\Pr,-\Pr)}. \tag{14}$$

RESULTS AND DISCUSSION

The ordinary differential equations (8) and (9) subject to the boundary conditions (10) have been solved numerically using an implicit finite-difference scheme developed by Keller, known as the Keller-box method, which is described in detail in the book by Cebeci and Bradshaw, (1988). This method has been successfully used by the present authors to solve various problems related to boundary layer flow and heat transfer (Bachok and Ishak, (2009 a, b), Bachok *et al.,* (2009) and Ishak *et al.,* (2010)). The results are given to carry out a parametric study showing influences of the curvature parameter γ and Prandtl number Pr on the fluid flow and heat transfer characteristics. For the validation of the numerical results obtained in this study, the case when the curvature parameter is absent ($\gamma=0$, flat plate) has also been considered and compared with previously published results available in the literature. Table 1 presents the numerical values of the surface temperature $\theta(0)$ along with the results reported by Elbashbeshy, (1998) and Liu, (2005) as well as the series solution given by Equation (14), and they are found to be in a very good agreement.

TABLE 1: Variations of $\theta(0)$ for different values of γ and Pr

γ	Pr	Elbashbeshy (1998)	Liu (2005)	Equation (14)	Numerical results
0	0.72	1.2253		1.236657472	1.2367
	1	1.0000		1.000000000	1.0000
	6.7		0.333303	0.3333030614	0.3333
	10	0.2688		0.2687685151	0.2688
1	0.72				0.8701
	1				0.7439
	6.7				0.2966
	10				0.2442

As shown in Figure 1, the skin friction coefficient $f''(0)$ is negative for all values of γ. Physically, negative value of $f''(0)$ means the surface exerts a drag force on the fluid, and the positive value means the opposite. This is not surprising since in the present problem, we consider the case of a stretching cylinder, which induces the flow. Since Figures (8) and (9) are uncoupled, then the Prandtl number Pr give no influence to the value of $f''(0)$, as can be seen in Figure 1. The absolute value of $f''(0)$ increases as γ increases, which in turn results in increasing manner of the heat transfer rate at the surface $1/\theta(0)$, for both $Pr = 0.7$ (air) and $Pr = 7$ (water). Thus, the skin friction as well as the heat transfer rate at the surface is larger for a cylinder compared to a flat plate, which was considered by Elbashbeshy, (1998) and Liu, (2005).

The velocity profiles for various values of γ are presented in Figure 2. This figure shows that the velocity gradient at the surface is larger for larger values of γ, which produces larger skin friction coefficient $|f''(0)|$, and this result is consistent with those presented in Figure 1. The temperature profiles depicted in Figures 3 and 4 also show that the temperature gradient at the surface increases as γ increases, which also agrees with those presented in Figure 1, i.e. the heat transfer rate at the surface $1/\theta(0)$ increases with an increase in γ. For fixed value of γ, the heat transfer rate increases with Pr , since the higher Prandtl number fluid has a lower thermal conductivity (or a higher viscosity) which results in thinner thermal boundary layer and hence, higher heat transfer rate at the surface (see Figure 5). Finally, Figures 2-5 show that the far field boundary conditions (10) are satisfied asymptotically, which support the validity of the numerical results obtained.

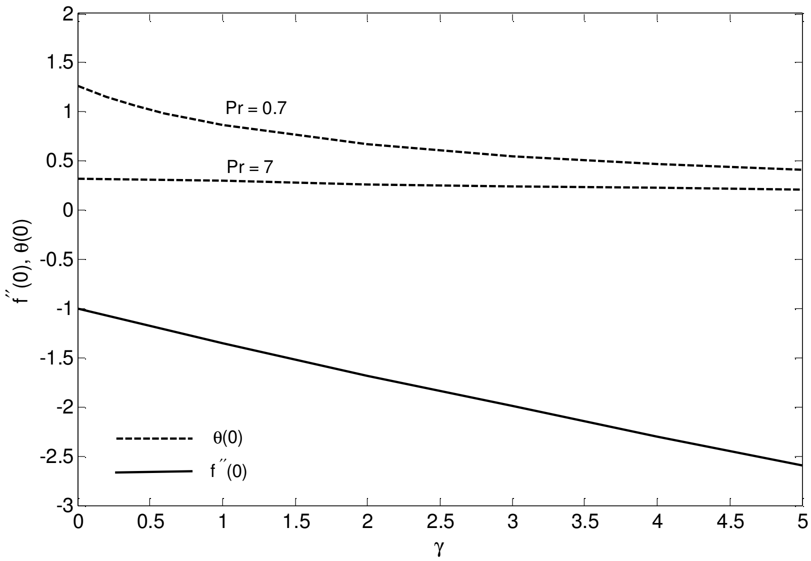

Figure 1: Variations of $f''(0)$ and $\theta(0)$ with γ for $Pr = 0.7$ and $Pr = 7$

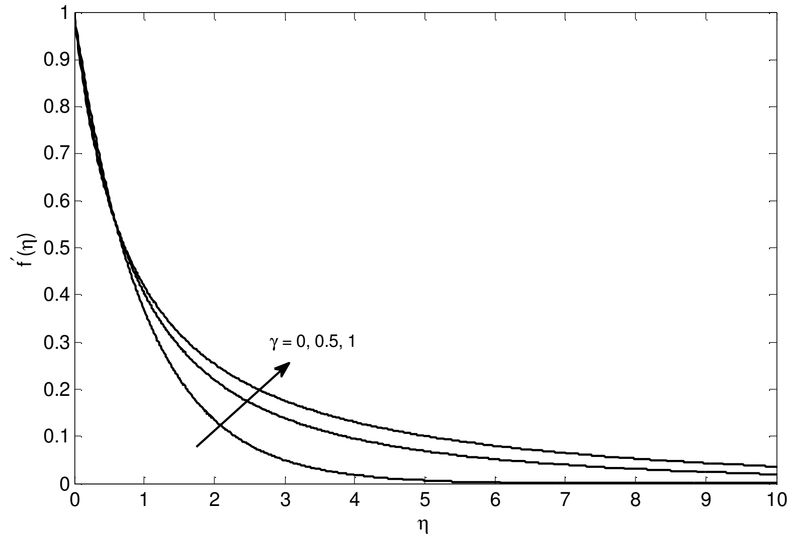

Figure 2: Velocity profiles $f'(\eta)$ for some values of γ

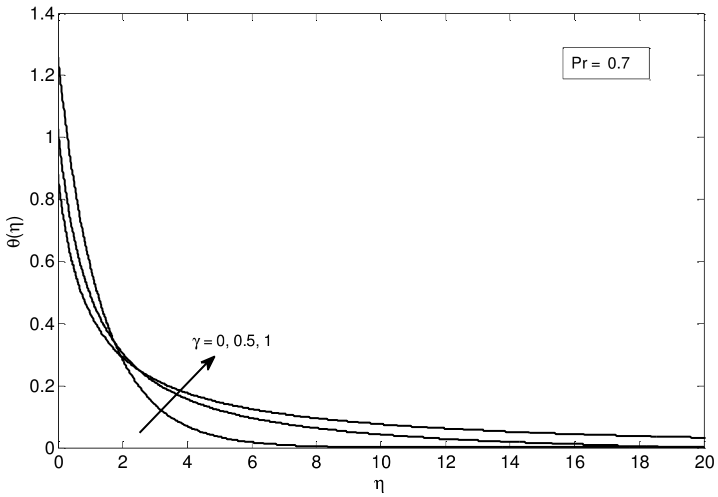

Figure 3: Temperature profiles $\theta(\eta)$ for some values of γ when Pr = 0.7

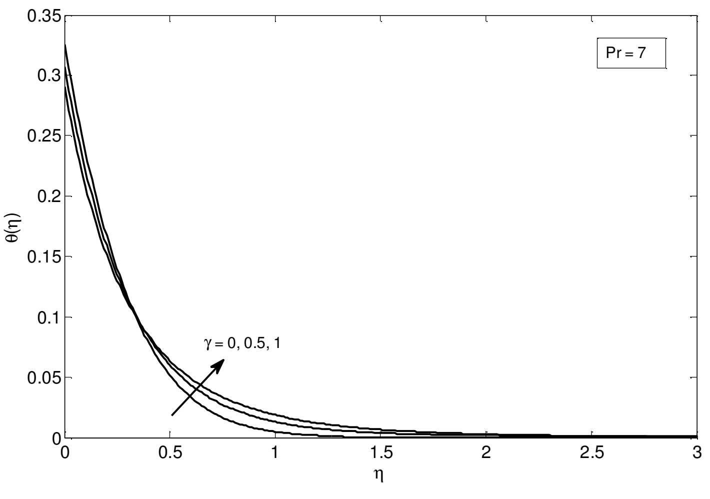

Figure 4: Temperature profiles $\theta(\eta)$ for some values of γ when $Pr = 7$

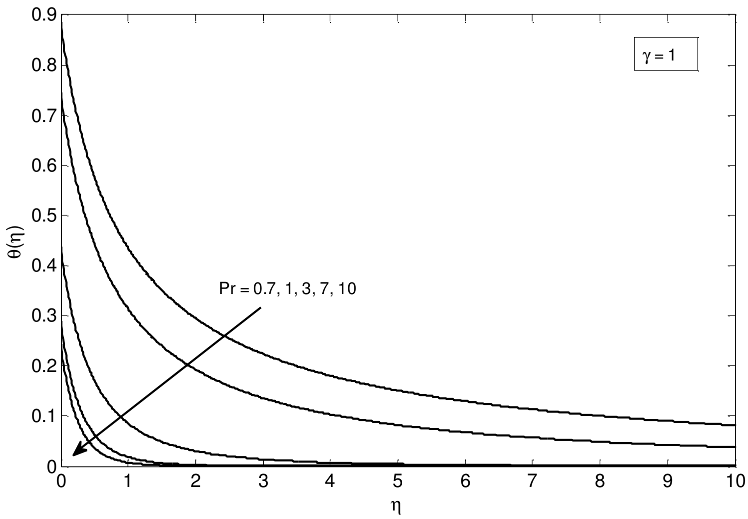

Figure 5: Temperature profiles $\theta(\eta)$ for some values of Pr when $\gamma = 1$

CONCLUSIONS

We have theoretically studied how the governing parameters, namely the curvature parameter γ and Prandtl number Pr influence the boundary layer flow and heat transfer characteristics on the surface of a horizontal cylinder. When $\gamma = 0$, the problem under consideration reduces to the flat plate case considered by the previous investigations. Further, the investigation on the effects of γ on the skin friction coefficient and the local Nusselt number reveals that both of them increase as γ increases. Thus, the surface shear stress and the heat transfer rate at the surface increase as the curvature parameter increases.

ACKNOWLEDGEMENT

The financial support received in the form of a fundamental research grant from Ministry of Higher Education Malaysia is gratefully acknowledged.

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